Some Lost Instabilities, 3D simulations, and SN2009ip

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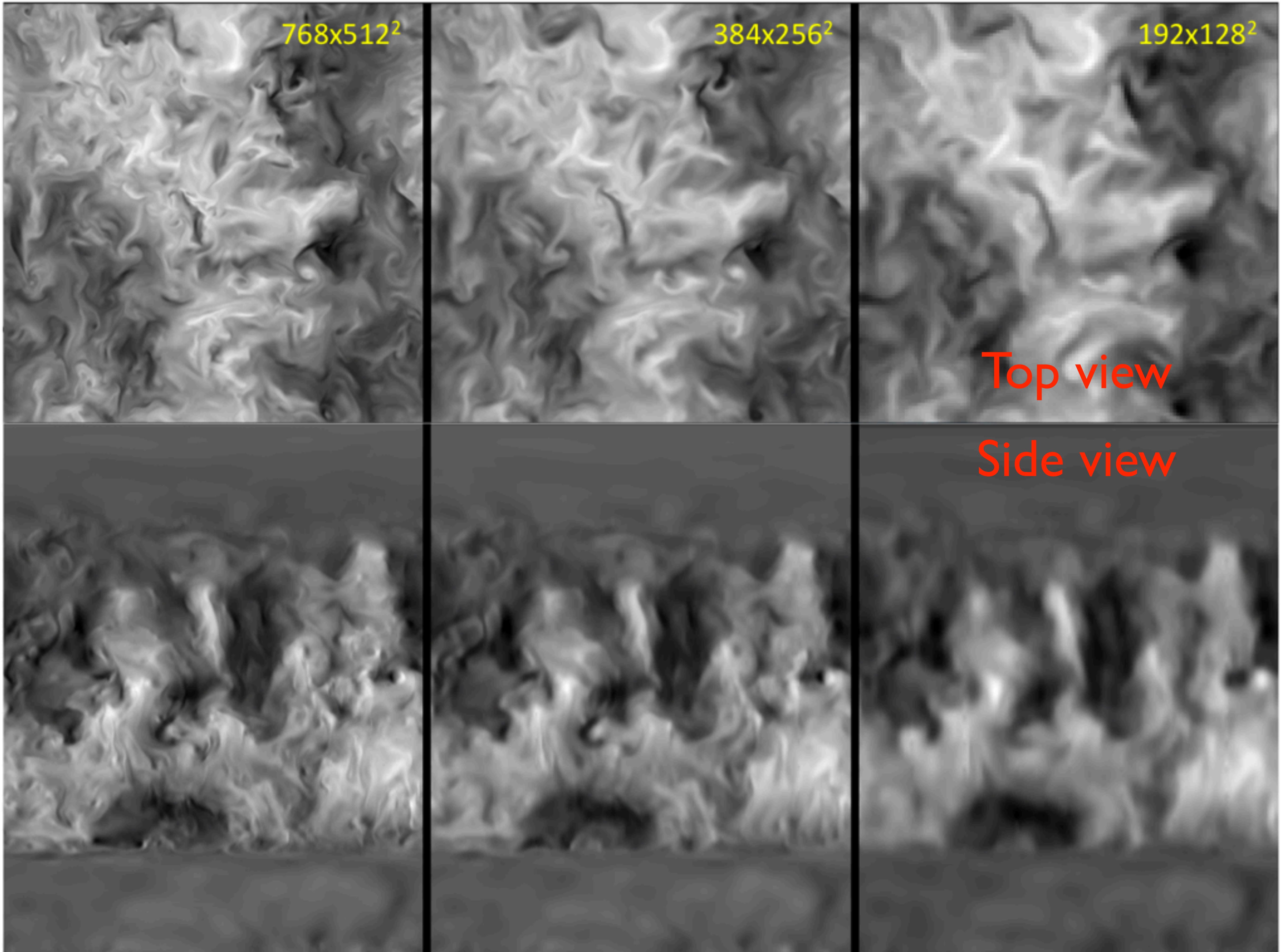
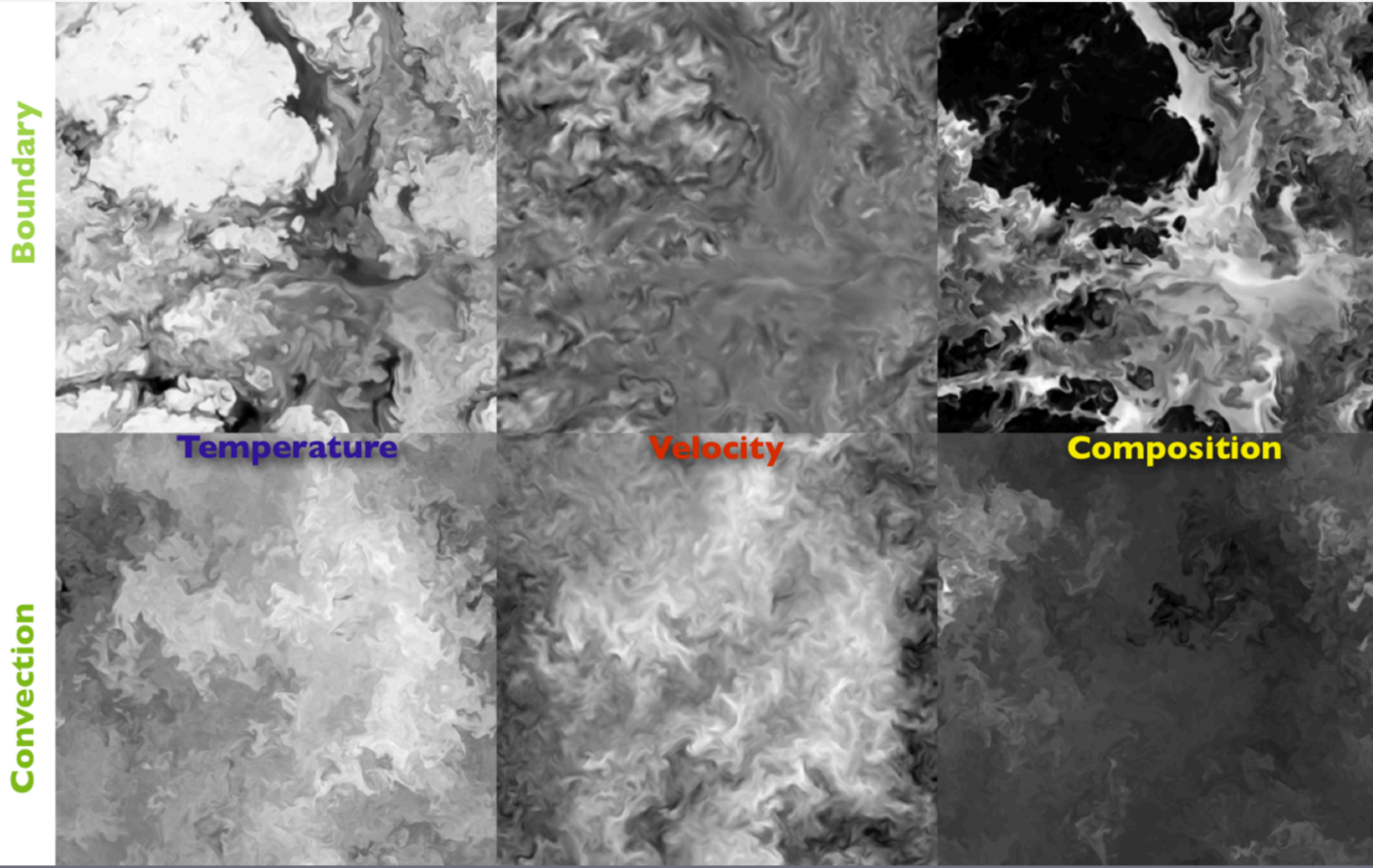
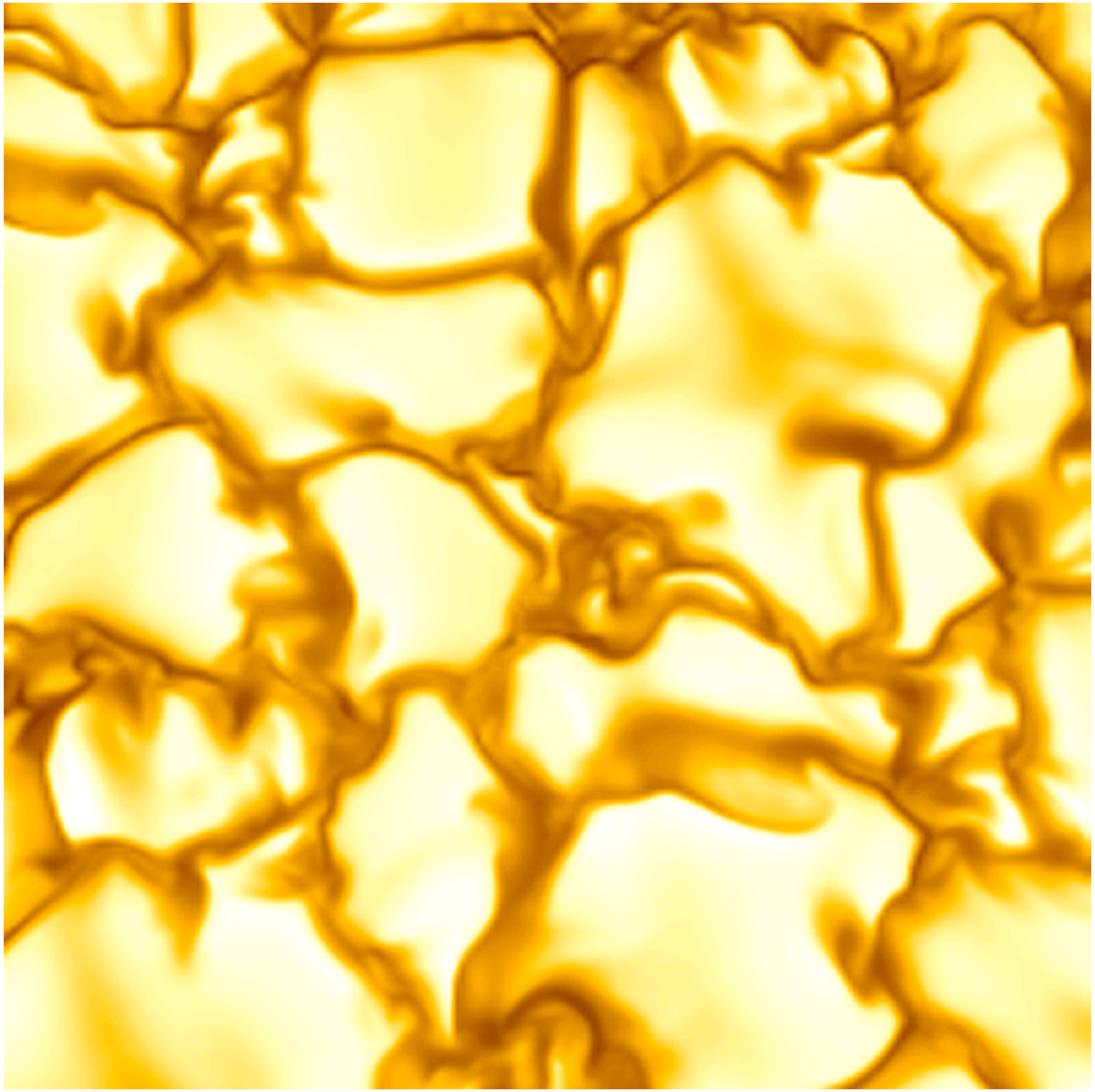
3D simulations of stars (nonexplosive)

- solar atmosphere (Nordlund & Stein, Stagger code, Freitag & Ludwig, CO5BOLD, Muthsam, Antares, ...)
- solar convective zone, rotation and MHD (Juri Toomre & Boulder group, ASH code, ...)
- global stellar fluid dynamics (Porter & Woodward, PPM code)
- deep convection (Meakin & Arnett, Prompi code and Viallet, MUSIC implicit code, Mueller, Mocak, Prometheus code, all multi-fluid)
- AGB mixing (Stancliffe, Lattanzio, Campbell, Djehuty code, multi-fluid)



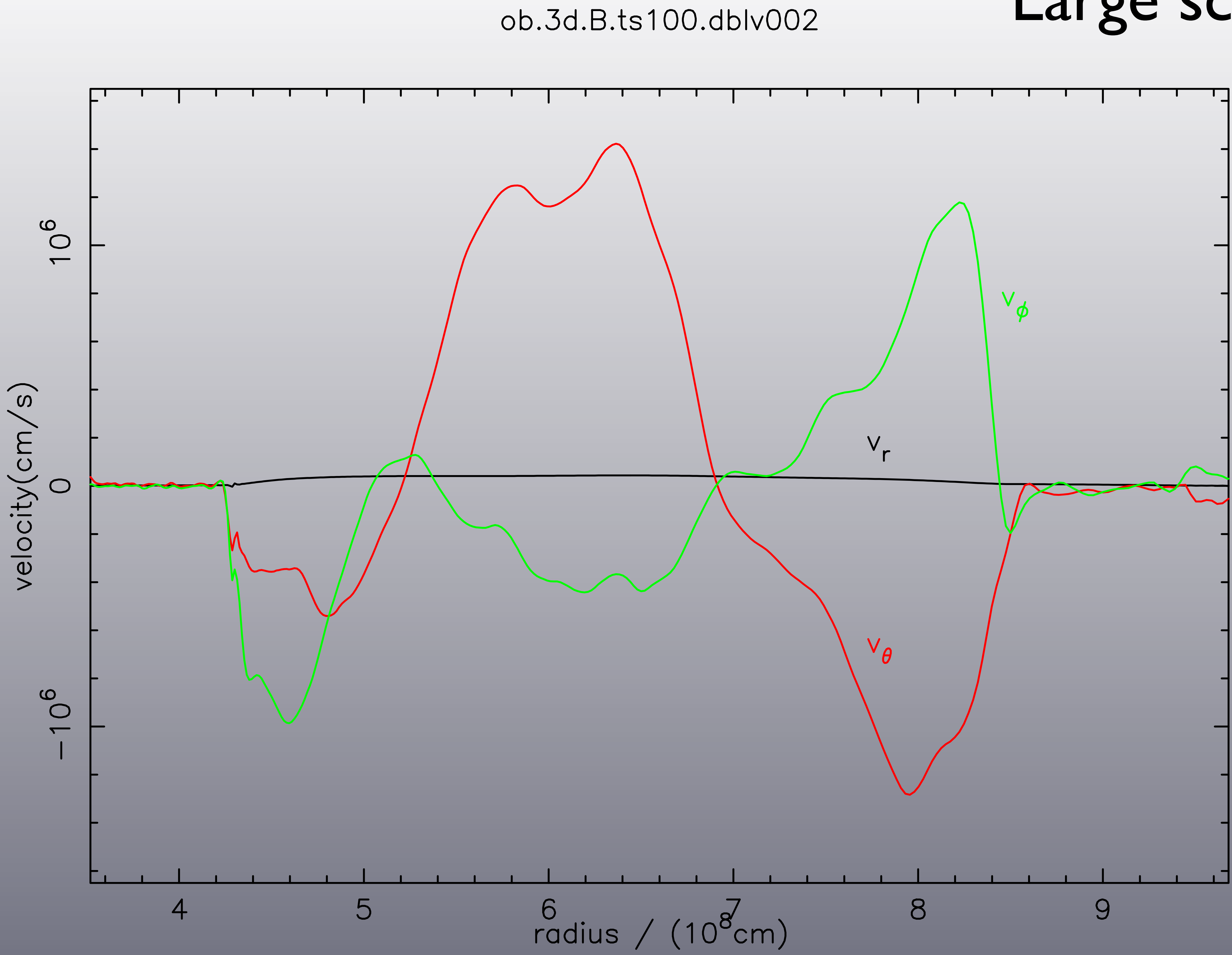
768x512^2
384x256^2
192x128^2
Top view
Side view


Boundary
Temperature
Velocity
Composition
Convection

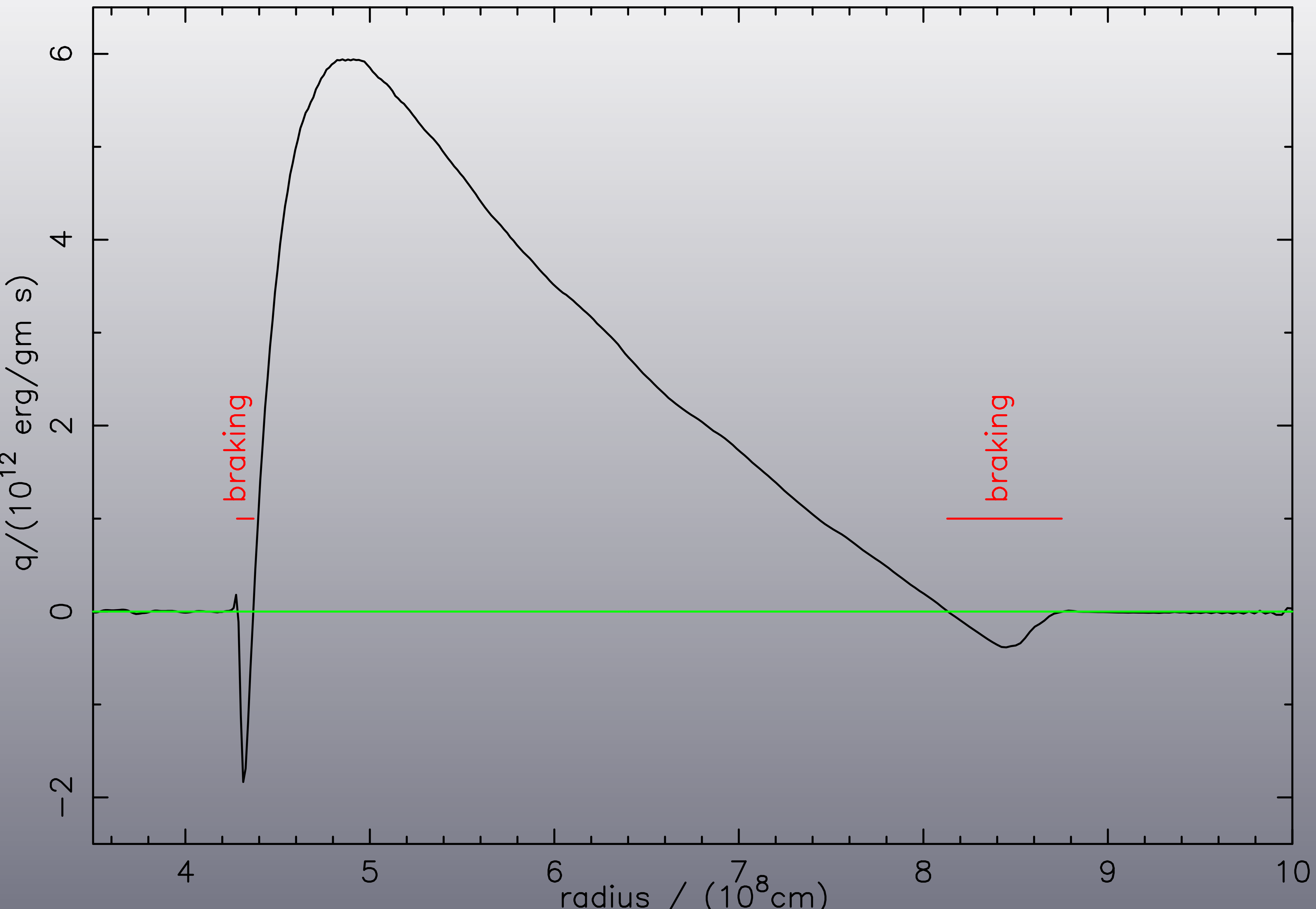


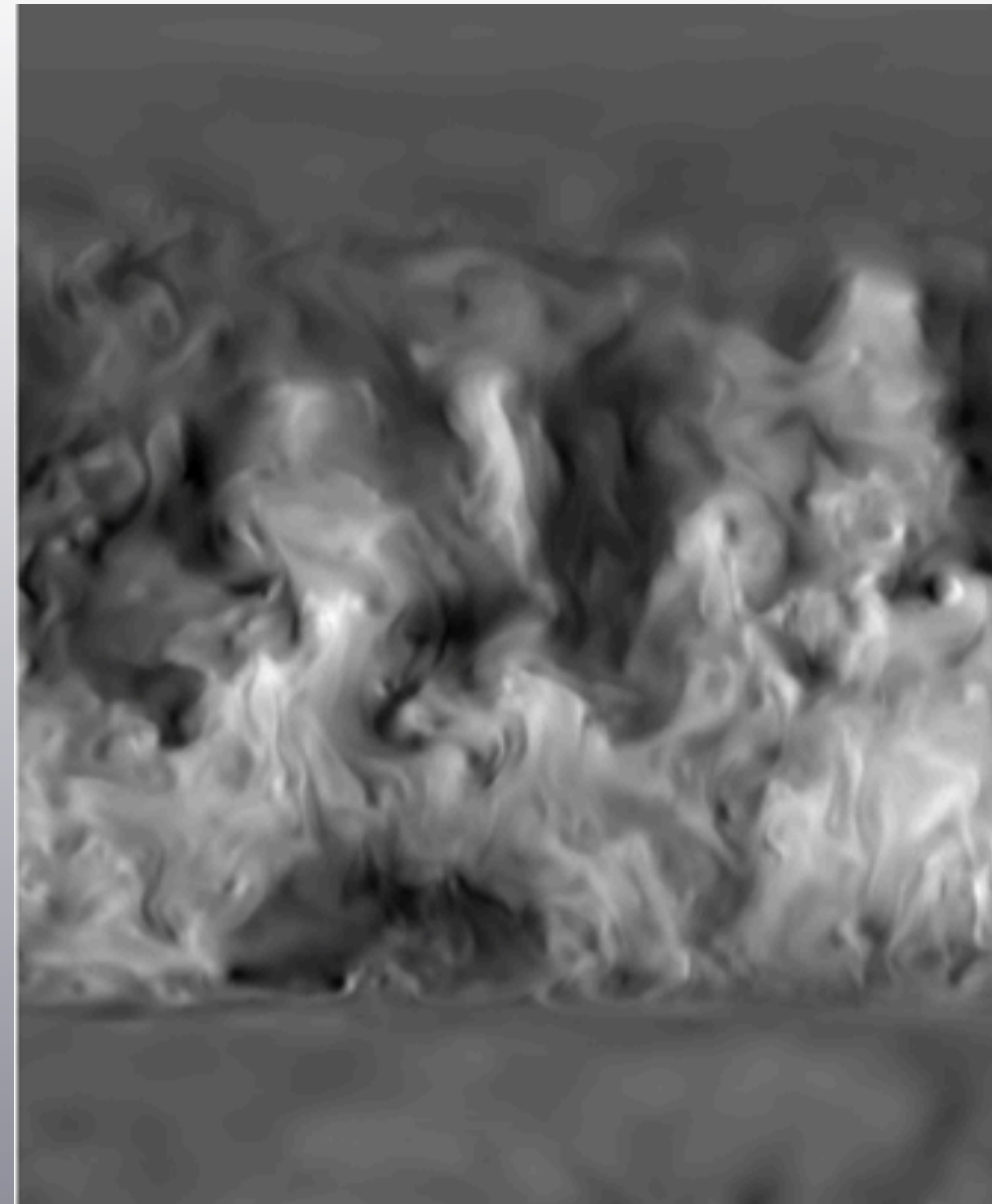
Nordlund & Stein: granulation geometry (flyover)

Large scale flows



Buoyancy averaged over angle and time, showing braking regions at boundaries





A snapshot of turbulent convection

Note quadratic nature of fluxes

$$q = \overline{\langle \mathbf{g} \cdot \mathbf{u}' \rho' \rangle} / \rho_0$$

Buoyancy

$$\mathbf{F_e} = \overline{\langle \rho \mathbf{C_P} \mathbf{u}' \mathbf{T}' \rangle}$$

Enthalpy

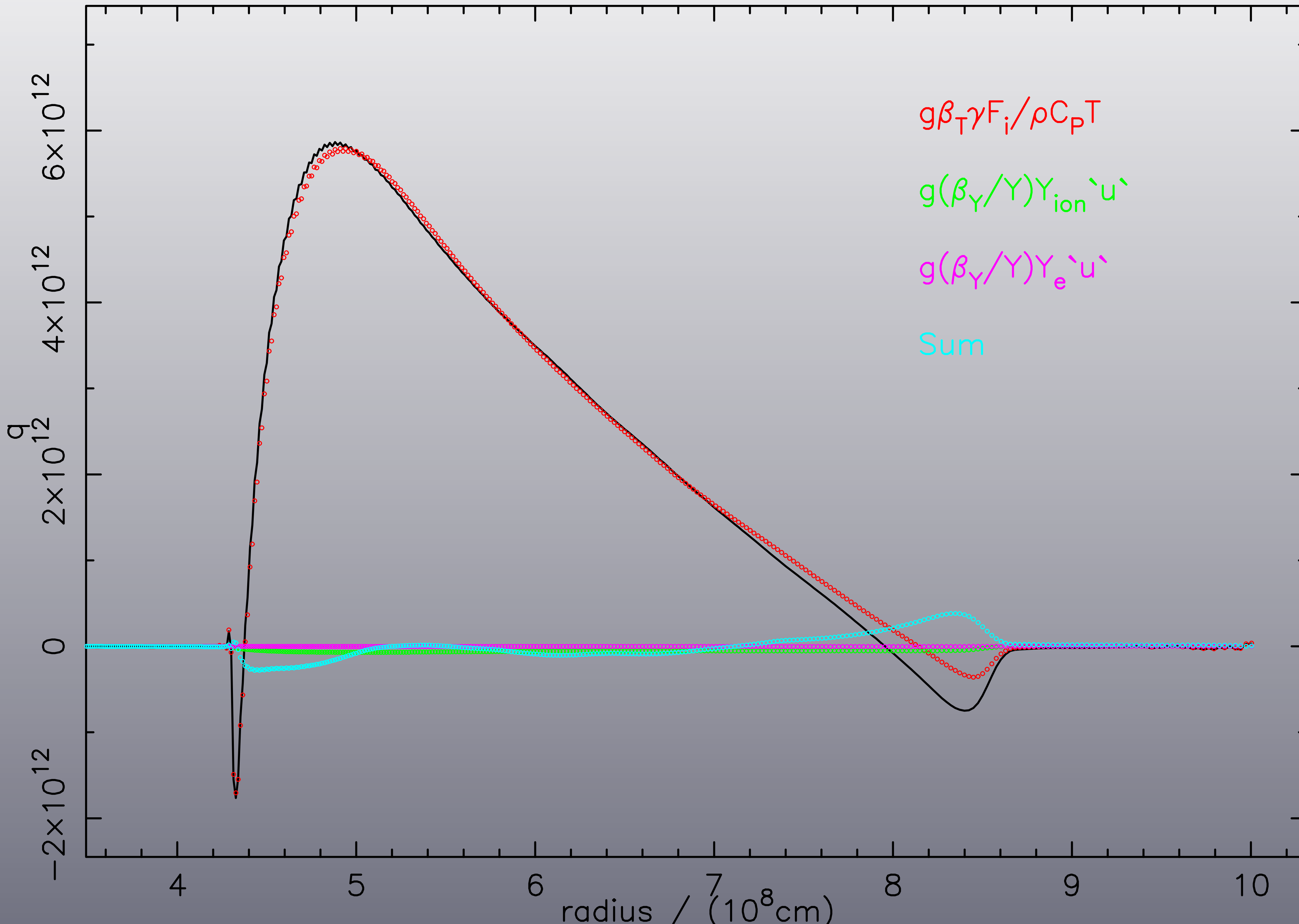
$$\mathbf{f_{Y_i}} = \overline{\langle \mathbf{u}' Y_i' \rangle}$$

Composition

For low mach number flow and a linear EOS

$$\overline{\langle \rho' u' \rangle} / \overline{\rho} = \beta_T \overline{\langle T' u' \rangle} / \overline{T} + \beta_Y \overline{\langle Y' u' \rangle} / \overline{Y}$$

Average over angle and time of fluxes of buoyancy, enthalpy, and composition (draft)



Turbulent kinetic energy equation

$$\begin{aligned}\overline{\langle\rho\rangle} D_t \overline{\langle E_K \rangle} &= -\nabla \cdot \overline{< \mathbf{F_p} + \mathbf{F_K} >} \\ &+ \overline{< \rho' \mathbf{g} \cdot \mathbf{u}' >} - \varepsilon_K\end{aligned}$$

Steady state, integrated over the convective zone

$$\int_{CZ} q \; dm = \frac{1}{\ell_d} \int_{CZ} \frac{\varepsilon_K}{\rho} dm$$

Kolmogorov's 4/5 law (homogeneous, isotropic)

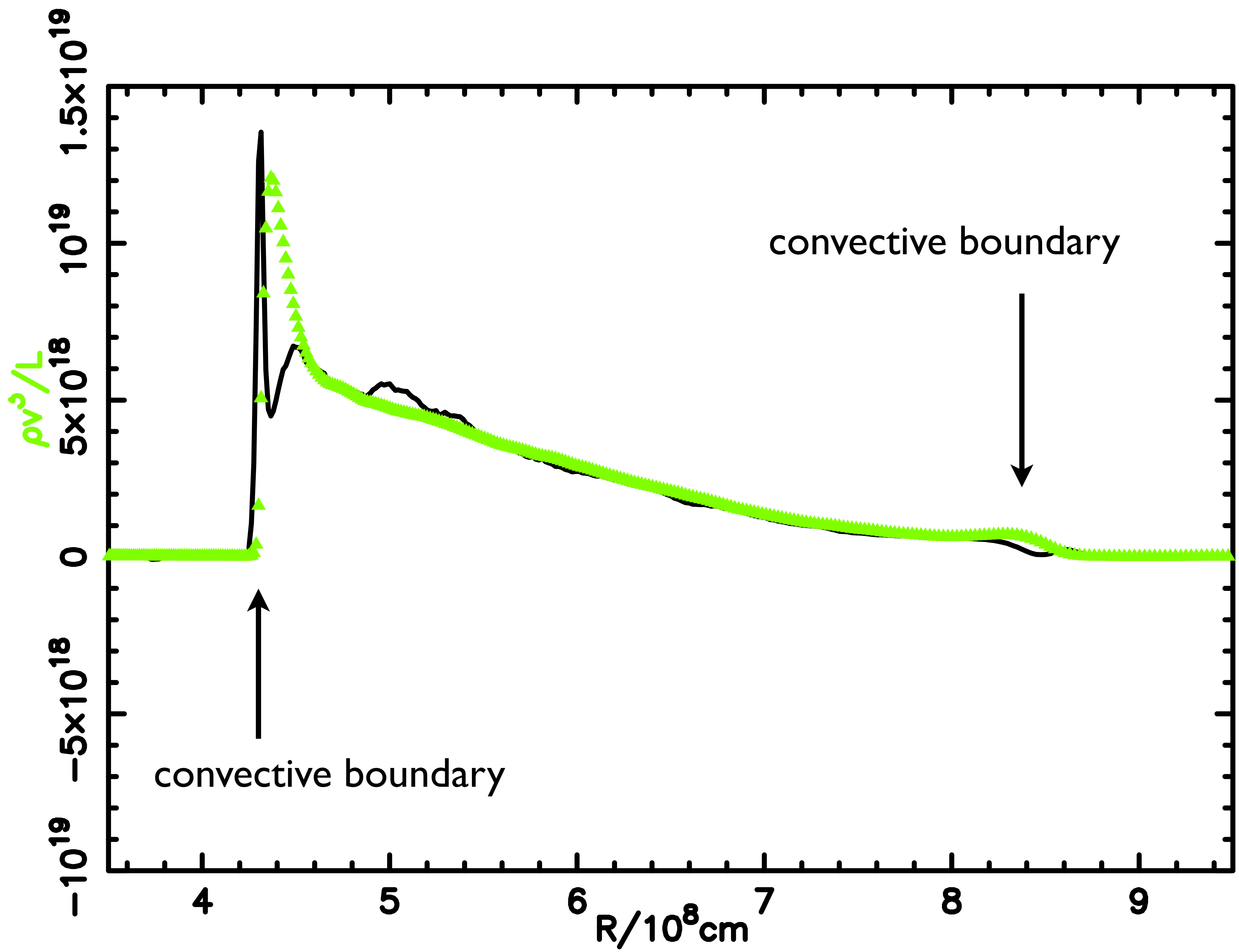
$$\varepsilon_K = \rho \langle |\Delta v|^3 \rangle / (\frac{4}{5} r)$$

Inhomogeneous, anisotropic case

$$\varepsilon_K = \rho v_{rms}^3 / \ell_d, \qquad \ell_d \approx (0.8 \text{ to } 0.9) r_{CZ}$$

This constrains the arbitrary choice of mixing length

Subgrid dissipation versus Kolmogorov



Radial acceleration equation, using buoyancy and Kolmogorov damping

$$\partial u/\partial t + u\partial u/\partial r = \beta_T g \Delta\nabla - u|u|/\ell_d$$

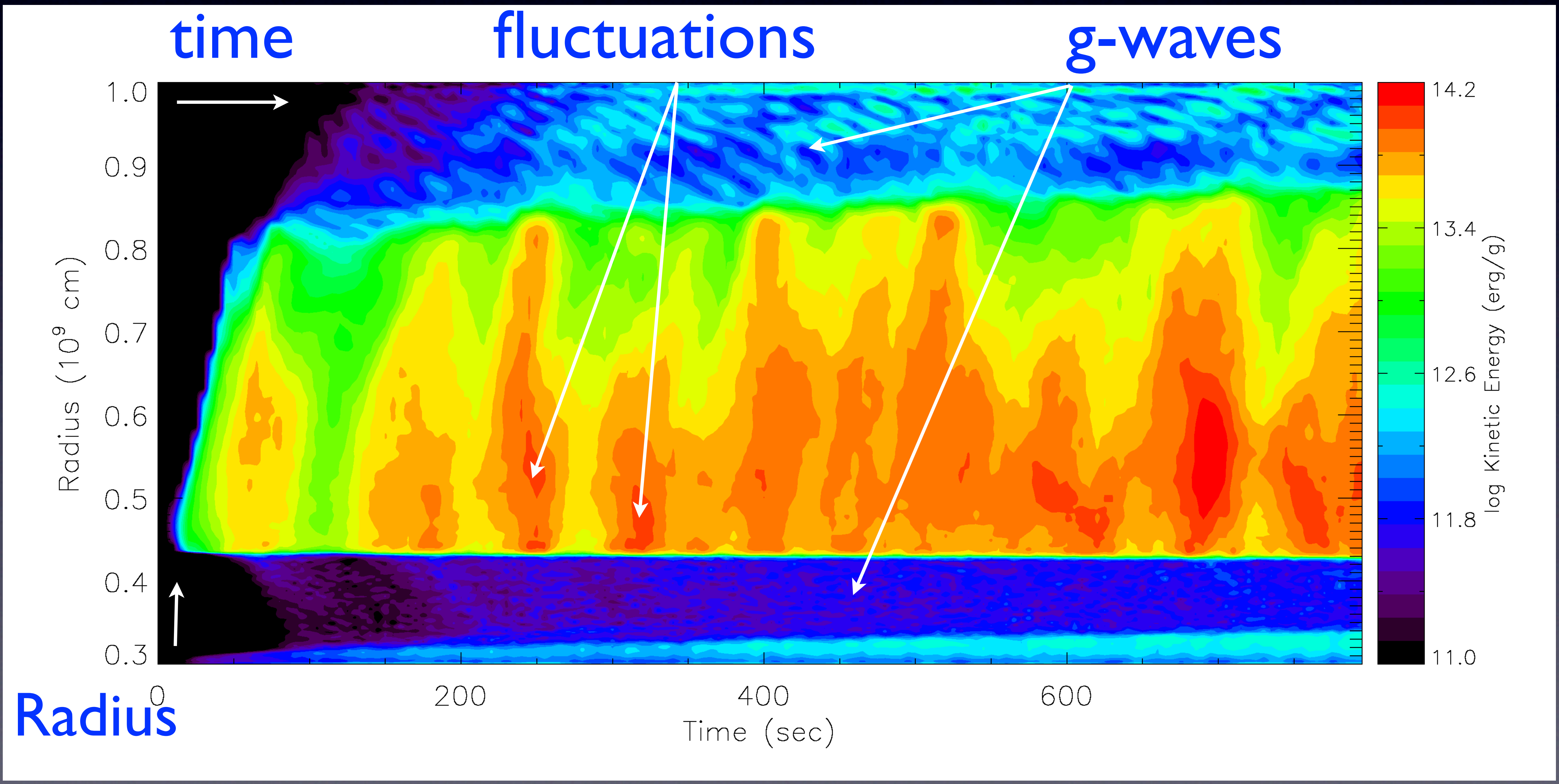
Near Convective boundary

$$\partial(u^2/2)/\partial r \approx \beta_T g \Delta\nabla < 0$$

a. gradient Richardson criterion for mixing
b. centrifugal force to reverse flow
c. negative delta nabla not allowed in MLT

This approach may be generalized (e.g., rotation and MHD)

KE fluctuations in Oxygen burning



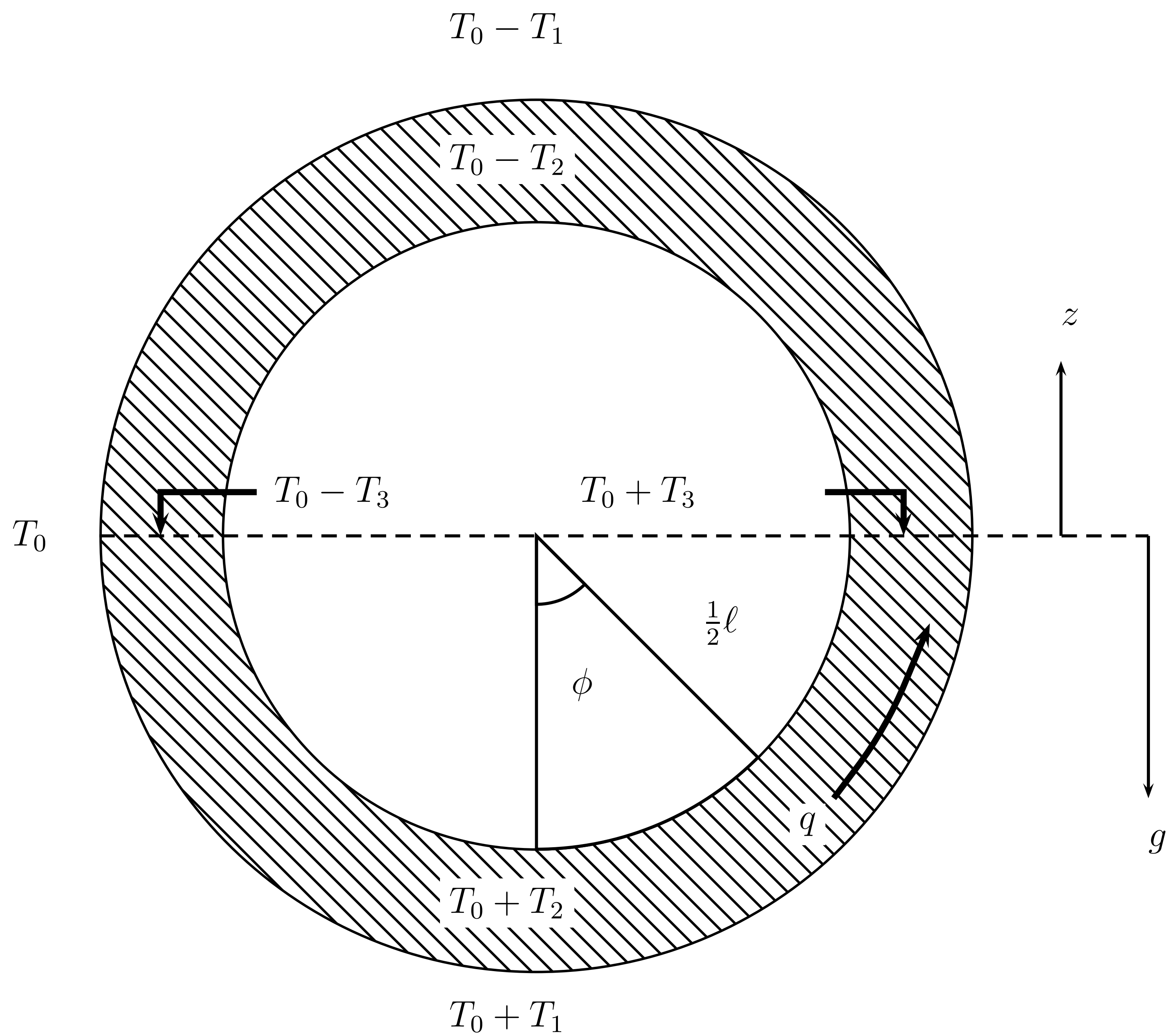


Fig. 3.— The Lorenz Model of Convection: Convection in a Loop.

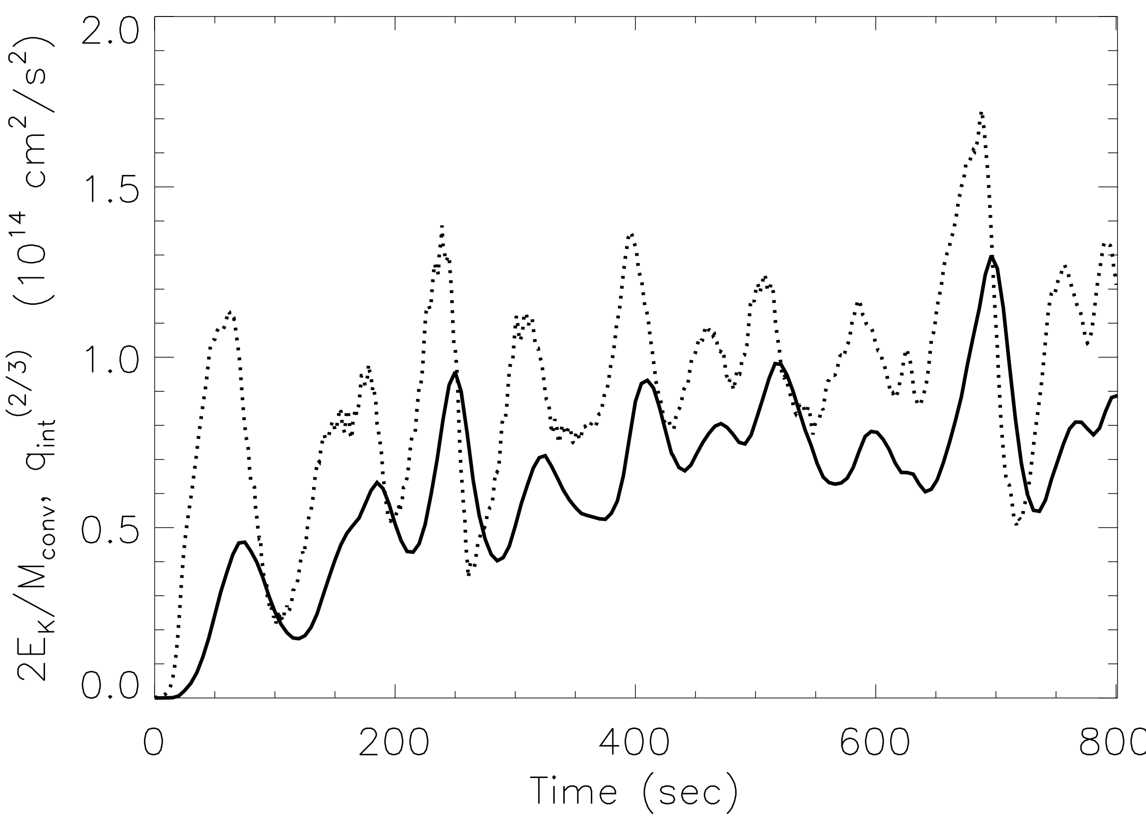
Lorenz equations:

$$
\begin{aligned}
dX/d\tau &= -\sigma X + \sigma Y \\
dY/d\tau &= -XZ + rX - Y \\
dZ/d\tau &= XY - bZ,
\end{aligned}
$$

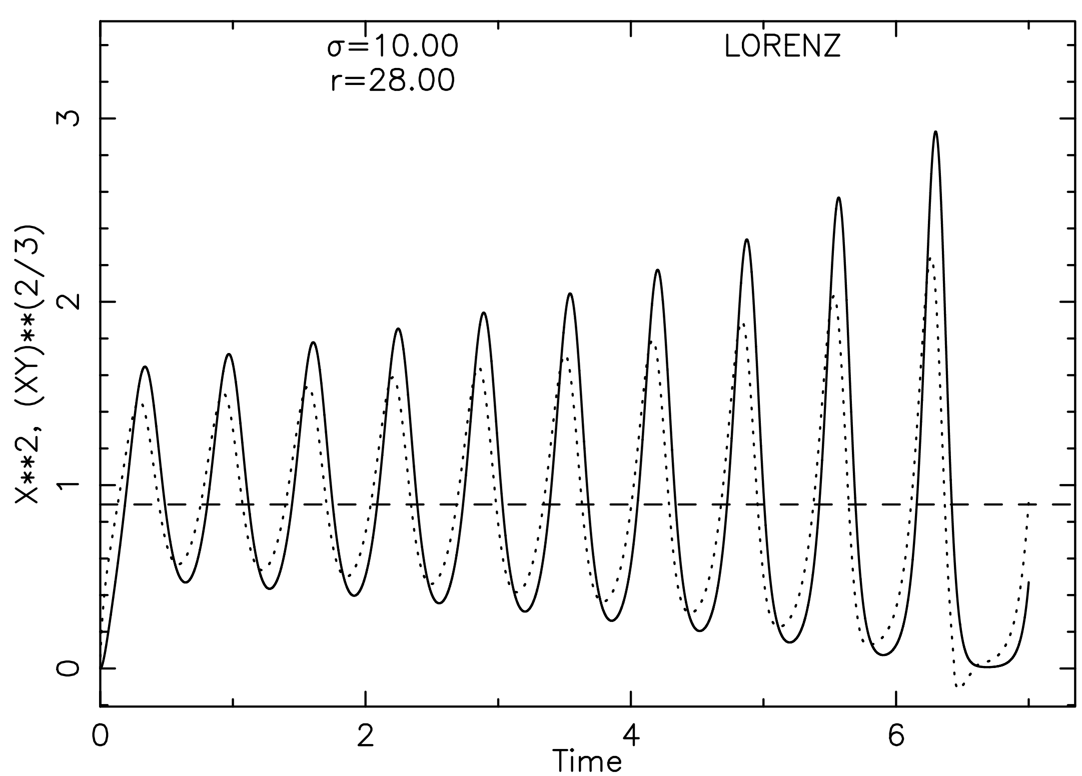
X: dimensionless speed
Y: dimensionless temperature difference (horizontal)
Z: dimensionless temperature difference (vertical)
r:Rayleigh No./critical, sigma: Prandtl No.
Time in units of radiative cooling time

Nonlinear instability

- Does not appear in linear stability analysis (Cox, Unno, etc.)
- Lorenz model has a strange attractor which is due to quadratic terms
- Turbulence can actively drive fluctuations in velocity and luminosity
- linear and nonlinear modes can couple, including pulsations, turbulence and nuclear burning



3D simulation



Lorenz model

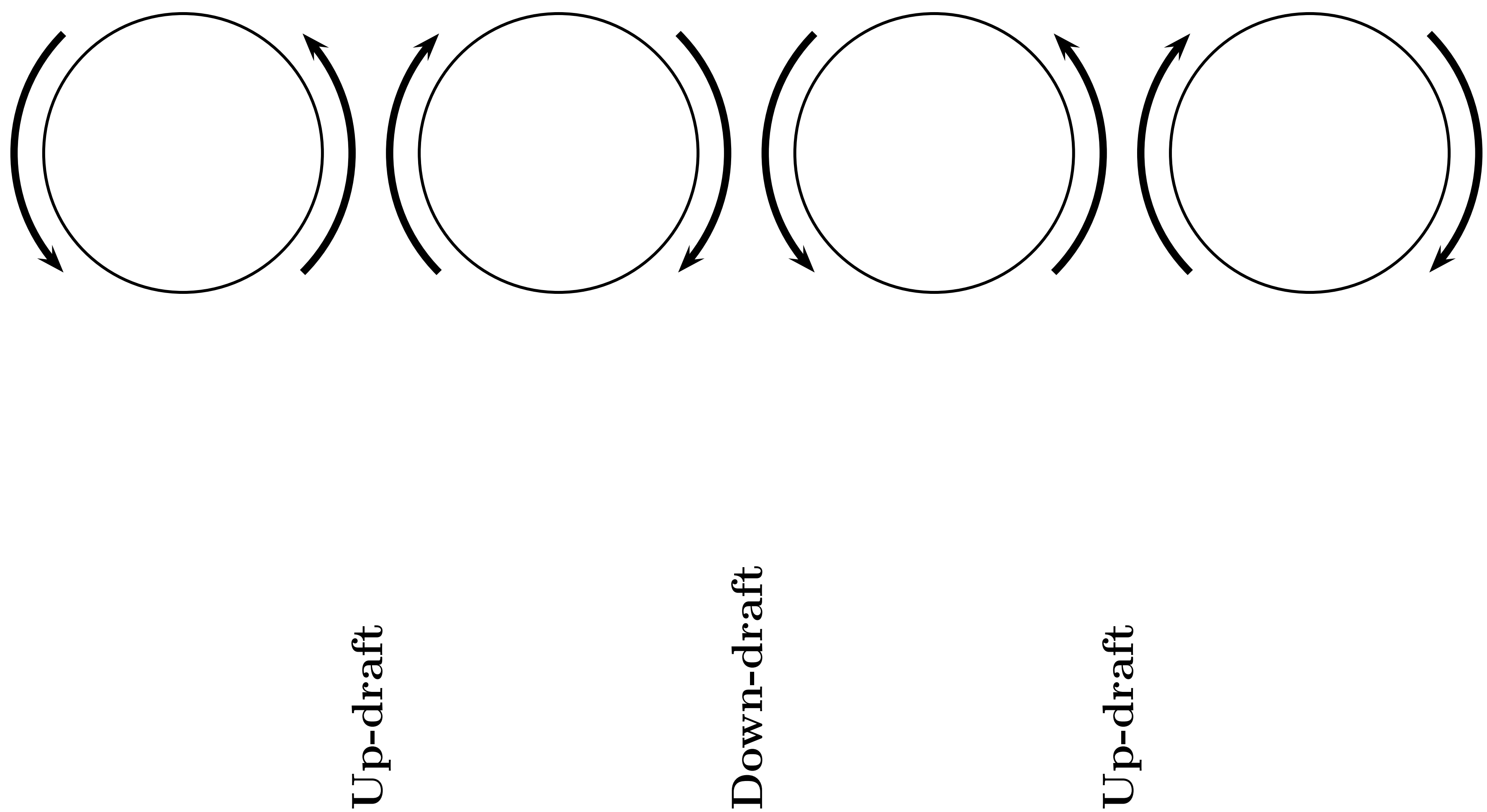
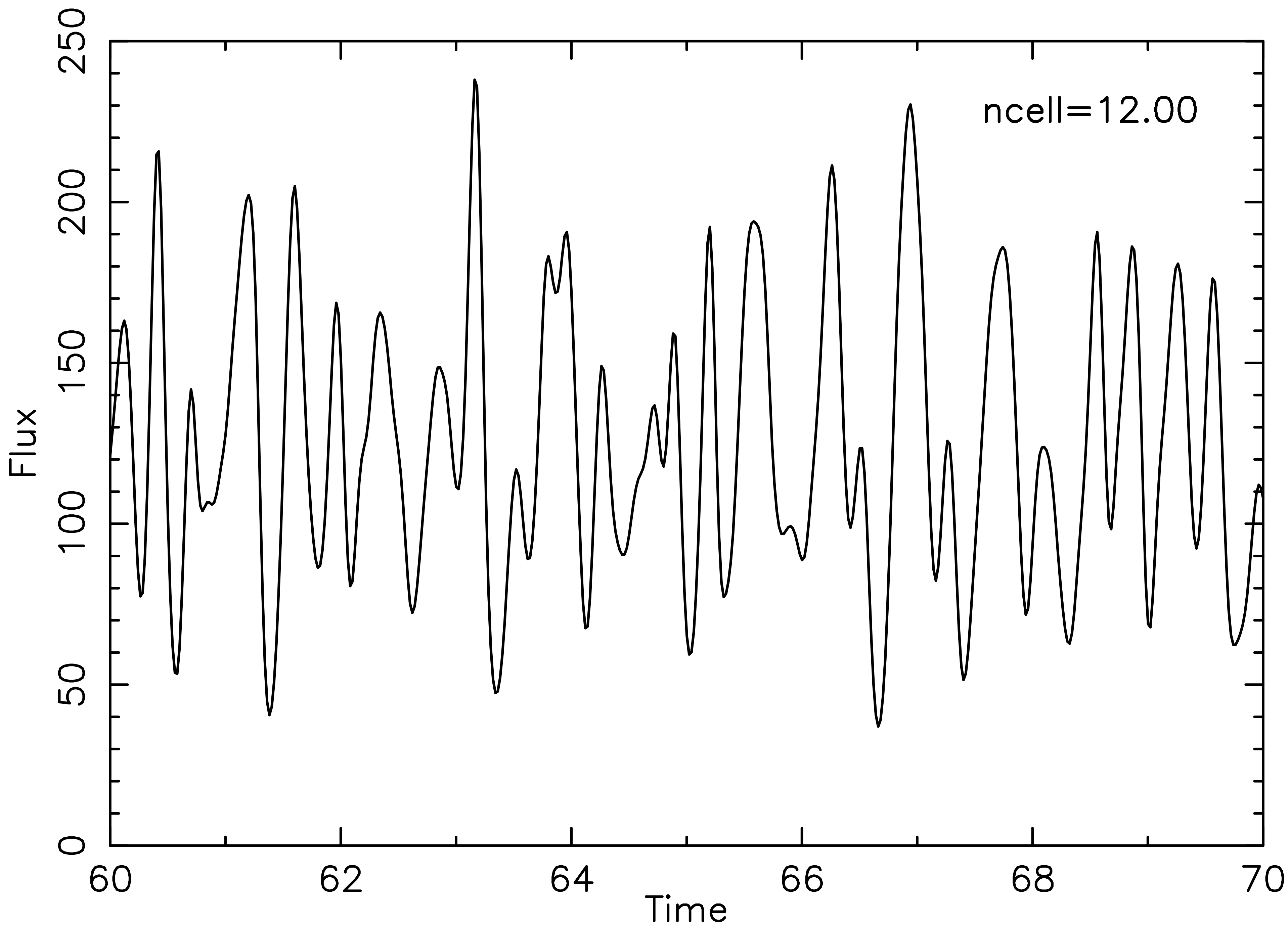
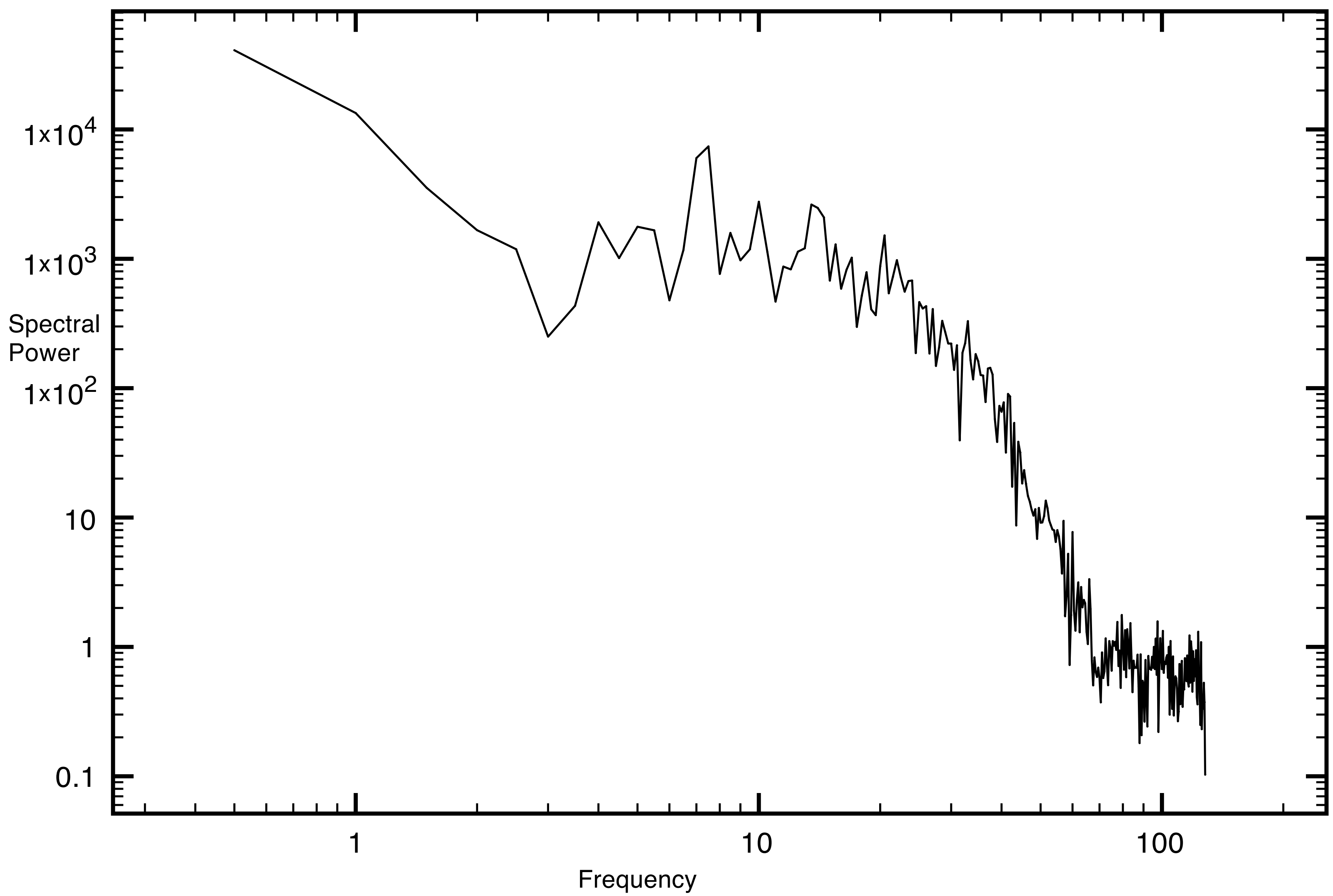


Fig. 4.— The Lorenz Model extended: Convection in a shell composed of cells. Notice the alternation of the sign of rotation. This may be thought of as a cross sectional view of infinitely long cylindrical rolls, or of a set of toroidal cells, with pairwise alternating vorticity. Each cell can exhibit random fluctuations in time.

Schwarzschild model using Lorenz : a fake Betelgeuse



Spectral power of Schwarzschild-Lorenz Fluctuations



Eruption prior to core collapse: 25 Msun (Meakin & Arnett, 2011, ApJ, 733, 78) 2D simulation

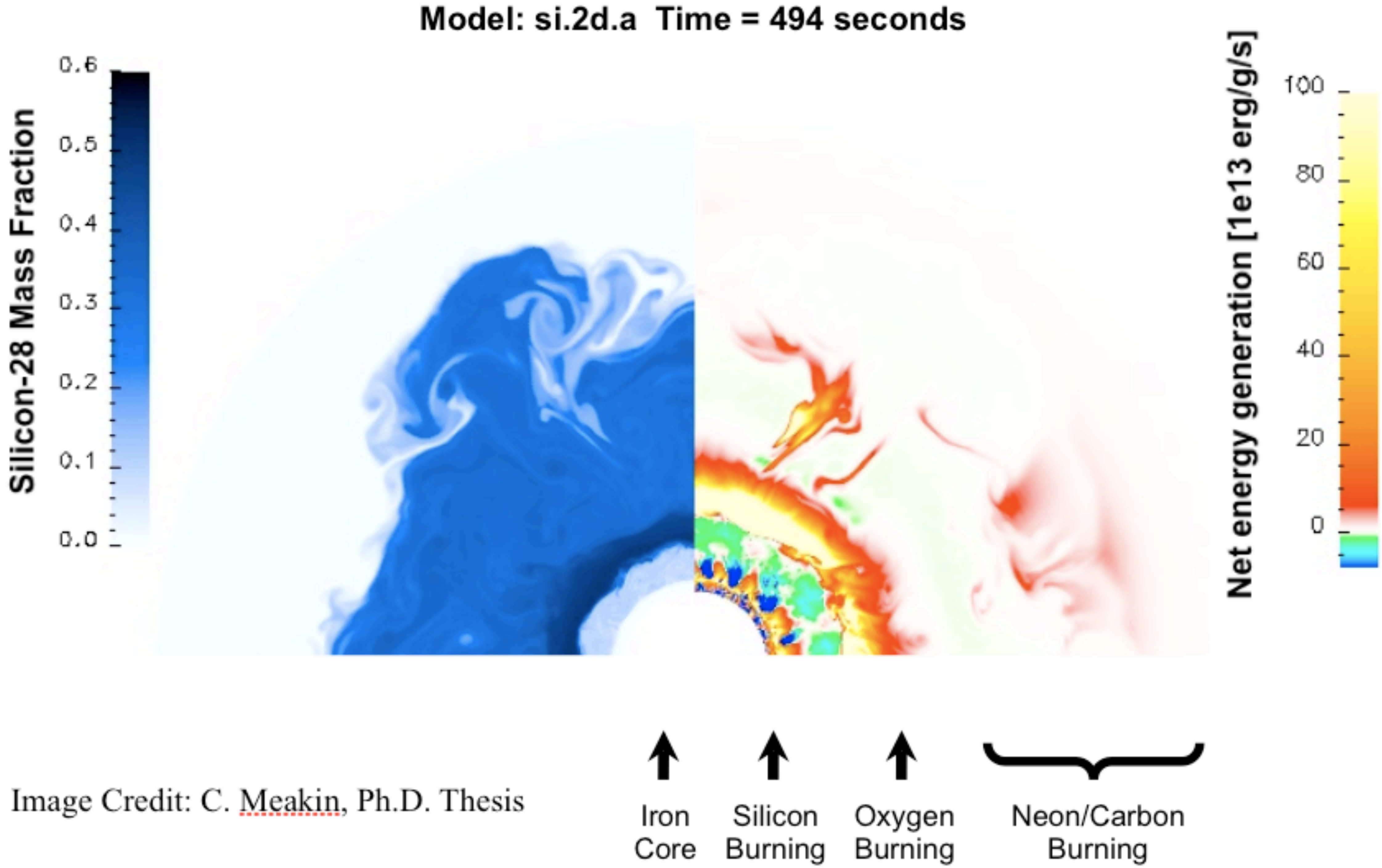


Image Credit: C. Meakin, Ph.D. Thesis

The progenitor problem

- 2D and 3D simulations having Oxygen, Neon and Carbon burning shells pulse but are quasi-stable
- 2D simulations including more advanced burning (Silicon) give eruptions prior to collapse
- Is this a 2D-3D effect?
- Is this an effect of advanced (extreme) evolution?
- Is this an effect of shell interaction (O and Si)?
- Jeremiah Murphy showed that progenitors are stable to linear instability, but these instabilities are nonlinear.
- We have just been granted sufficient computer resources to find out.

Summary

- Linear stability analysis is wrong for convection, because turbulence in nonlinear (e.g., Lorenz model)
- 3D simulations of stars are now able to reproduce turbulent flow, and show a balance between buoyancy and turbulent damping, allowing the "MLT parameter" (velocity scale) to be strongly constrained
- Stellar convection zones have turbulent braking layers (not possible in MLT)
- A simple dynamic model for the largest eddies includes Kolmogorov cascade, Richardson criterion, non-locality
- Turbulent fluctuations can and probably do drive unrecognized instabilities in stellar evolution, especially in massive stars near collapse
- Fundamental problems remain (e.g., ILES values of Prandtl number and entrainment)